Spreadsheet Based Application of The ‘Standard Model’
In Variable Part Thickness Radiography

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ABSTRACT
The ‘standard model’ of radiographic exposure technique allows for generation of exposure technique variables in various forms. A Microsoft Excel spreadsheet application and additional dedicated ‘Add-In’ functions are described that implement the standard model.

The Visual Basic for Applications (VBA) coded application (named ‘StandardXp’) is a menu driven system and allows for traditional fixed kVp and variable kVp method solutions as well as the variable source to image distance (SID) method and also ‘combination method’ (simultaneous variation of tube voltage and tube charge) types of solution. The associated set of ‘Add-In’ functions also enable the same solutions (using any of the methods just mentioned), to be obtained. Aspects of implementation of the standard model in the coded calculations are described, and some limitations of the StandardXp application and functions in generating usable exposure charts are also discussed. Such limitations are suggested to be mainly due to limitations of the standard model itself.

Keywords:
Radiographic Exposimetry, Radiographic Exposure, Exposure Technique, Radiological Physics, Spreadsheet Applications.

INTRODUCTION
The traditional approaches to radiographic exposure manipulation in order to compensate for a varying part thickness (in order to keep the average image density constant) are known as: ‘fixed kVp’, ‘optimal kVp’ (similar to the fixed kVp approach), ‘variable kVp’ and also the lesser known and used ‘variable source to image distance (SID)’.

These approaches have been well described in various texts and journals (eg. Kelly, 1988, Carlton and Adler, 1996, and Fuchs, 1955). The author of the present article has previously shown that these methods are equivalent to the use of a simple model of image density (Garbett, 1999), a model henceforth referred to as the ‘standard model’.

The standard model itself also contains solutions to the problem of variable part thickness radiography that represent a ‘combined approach’ (see Garbett, 1999 for a full description of such an approach). Since maintaining a constant SID value will lead to a constant magnification value, a combination approach that involves only variation of the passed tube charge (mAs) and of tube voltage (kV) is generally preferable to one involving also variation of the SID.

Terms used in this article
Below are definitions of some terms used in this article, these may be somewhat uncommon due to the nature of the material. Where a symbol has also been conveniently used for that term then this is also given alongside the worded term.

(i) ‘Tube charge’, \( q \).

As used in this article, this term refers to the product of tube current multiplied by time of irradiation, and can be measured in the common units of mA x s, or mAs, and so also in units of mC (milliCoulombs). It refers only to the total electrical charge passed between cathode and anode of the X ray tube.

(ii) ‘Tube Voltage’, \( V \).
Likewise this term refers only to the voltage applied between the anode and cathode of the tube for the duration of the exposure. The implicit assumption throughout this article (and also of the standard model) is that the value of \( V \) is constant (as produced ideally within a constant potential generator) and that it is measured in units of kilovolts (kV). Since the peak kilovoltage is then identical to this constant value, the symbolisation ‘kVp’ is then logically made redundant.

(iii) ‘Part Thickness’, \( x \).
This refers to the thickness of the part being radiographed. The ‘part’ may be part of a patient, or in deed any radiographed material with the property that the effective beam half value thickness (HVT) assumed to apply is 3.0 cm of the part material (an assumption of the standard model, see Garbett 1999).

(iv) ‘Approach solution’.
Where a given ‘approach’ (either a ‘fixed kVp’, variable kVp, variable SID, or ‘combination approach’ (variation of \( q \) and \( V \) together)) is assumed, then a knowledge of the variation of the operational variable/s involved with the part thickness, \( x \), is termed a solution (as traditionally understood in mathematics). For example, an approach solution assuming the fixed kVp technique refers to knowledge of the values of \( V \) (the operational variable in this case) with \( x \). Such approach solutions can be represented graphically, in tabular form, or by an equation.
say \( x_\text{e} \) (eg 20cm) then one may desire to know the values of \( q \) for various \( x \) from \( x_\text{e} \) up to 30cm. In this case 30 cm will be the ‘end thickness’. The end thickness can be smaller in value than the reference thickness so the possible intervals of thickness for a solution are always \( x_\text{e} < x < x_\text{f} \) or \( x < x_\text{e} \).

(v) VBA

VBA is an acronym for Visual Basic for Applications and refers to the contemporary incarnation of the BASIC computer programming language in the context of the Microsoft Office Applications suite using a Microsoft Windows based operating system. VBA has been traditionally regarded as a subset of the Visual Basic (VB) language which is not limited in context to the various Microsoft Office Applications.

Rationale for Spreadsheet Implementation

Manual exposure methods (as opposed to automatic methods employing ionization chambers near the image receptor) have often been presented as easily remembered rules (eg. the 25 per cent per cm rule for the fixed kV method) (eg Kelly, 1988), by some methodology such as the Siemens point system (Siemens, 1962), or alternatively by some combination of methodology as represented in tabular form (eg Kelly, 1988).

The common and widespread use of personal computers (PCs) in imaging departments and also educational establishments lends itself also to a computer driven implementation of exposure technique rules without any calculation on the part of the user, in the design of exposure charts utilising such rules.

As a means to the easy implementation of the standard model for the generation of exposure technique charts, an application using Microsoft Excel was developed using VBA coding. This allows a menu driven selection of any of the approach- es/methods that are consistent with the standard model, requiring no calculation on the part of the user. In addition to the workbook application, an Excel Add-In file comprised of four dedicated functions allowing the various approach solutions to be obtained, is also available.

The application (named ‘StandardXp’), utilises the Microsoft Excel workbook (.xls) file format as its basis (Excel is effectively the ‘parent application’), and this is convenient for most users because of the widespread availability of the Microsoft Office suite within the Windows operating systems. The Excel Add-In ‘StandXpFunc’ (.xla file format) allows additional Excel functions to be made available within any Excel session. These two files are not interdependent in any way and the user is free to install both or just one file.

Although Excel needs to be loaded to run StandardXp, no familiarity with Excel itself is required due to the menu driven structure of the application. On the other hand, usage of the add in functions does require general familiarity with the process of function calculation within Excel.

**BASIC DESCRIPTION**

(i) Workbook Application

StandardXp is a protected Excel workbook that contains VBA coded procedures (also known as macros), and two versions are available for Excel 5.0 and Excel 7.0. It can be easily loaded and accessed by using a desktop shortcut (as one option).

It is freely available for download by internet link from the authors web page: http://athene.riv.csu.edu.au/~igarbett/index.htm

and comes with documentation in a self extracting compressed (.exe) package.

The menu driven workbook application allows for the following approach methodology (also referred to as ‘solutions’ henceforth):

(a) ‘Fixed kVp’ solutions
(b) ‘Variable kVp’ solutions
(c) ‘Variable SID’ solutions
(d) ‘Combination Approach’ solutions utilising variation of tube voltage and tube charge together.

(ii) Add In File Functions

The separate Excel Add-In file ‘StandXPFunc.xla’ is also available within the same self extracting compressed (.exe) package together with appropriate documentation.

Eight functions are available. There are two functions for each solution-approach listed above, one function type being simple scalar and the other type an ‘array’ function. A scalar function returns only one value of the control variable at one specified ‘end thickness’ value, \( x_\text{end} \). An array function returns several values of the control variable together with \( x \) in an array form. The reader should refer to the descriptions found in Appendix I to properly understand the nature of these functions. For the time being the functions are merely identified and listed below:

- \( fL \)
- \( aL \)
- \( fV \)
- \( aV \)
- \( fq \)
- \( aq \)

The remainder of this article details several aspects involving both the workbook application and the Excel Add In functions’ uses and limitations.

**Using StandardXp: General**

StandardXp is opened in the same manner as any Excel workbook and is easily performed by the user having some familiarity with the Windows environment. Once Excel itself is loaded the file StandardXp.xls is then simply opened from within Excel.

Because the file itself contains VBA coded procedures, a message box warning will alert the user that the file contains ‘macros’ and whether the user wishes these to be enabled.

The macros need to be enabled for the application to run. A main menu sheet is then opened and allows for ‘point and click’ selection of any of the four solution methods cited above. The solutions are generated as a table and a graph of either \( q \) versus a selectable range of part thickness, \( x \) (cm).

It should also be noted that the application is protected to disallow alteration (inadvertently or otherwise) of many features, including the contained VBA coded procedures.

**Generating a ‘fixed KV’, ‘Variable KV’ or ‘Variable SID’ Solution**

The following description is specifically for a fixed kV solution (where the operator controlled variable is \( q \)), however implementation of either a variable kVp or variable SID approach solution is easily performed by simple substitution of the named control variables \( V \) or \( L \) respectively in the following description.

After selection of the ‘fixed kV’ method, clicking the ‘start'
The button implements the sequence of input boxes asking the user for:
(a) the value of known tube charge, \( q \), in mAs, (at a known part thickness)
(b) the value of that known thickness (in cm)
(c) the thickness you wish to compensate to (the ‘end thickness’ as previously defined)

Upon appropriate typed entry by the user of these values, a table listing \( q \) versus \( x \) (thickness) is generated, along with a plot of the values (the plot only is limited to ~ 96 cm thickness range).

The chosen ‘end thickness’ can be smaller or larger than the known reference technique thickness, but negative thickness entries are not allowed. Similarly only positive values of mAs can be entered for the reference technique. As in any conventional method of exposure chart generation, it is necessary to establish the ‘reference technique’ values by prior knowledge, this gained typically by use of a suitable radiological phantom for the particular system that is being used. Of course, as implied by these methods, only the single control quantity (e.g. \( q \) in a fixed kV method), and \( x \), are variables, the other exposure control quantities are assumed and required to be held constant.

Generating a ‘Combination (V and q) solution’
Selecting this option from the main menu (also by other menu buttons elsewhere in the application) displays an Excel worksheet that is similar in format to that presented for the other methods available.

After clicking the ‘start’ button, a series of input boxes are presented to the user for typed entry of values.
These require input of:
(a) Value of tube charge, \( q \) (in mAs) at a known (reference) thickness.
(b) Value of tube voltage (in kV) at that same thickness.
(c) Value of the reference thickness.
(d) The value of ‘end thickness’ (to which one is ‘compensating to’, in cm).

(e) The value of tube voltage (in kV) desired at the end thickness.
A table is then generated listing the control variables (here both \( V \) and \( q \)) versus the part thickness, \( x \), in increments of 1cm. A plot is also generated showing variation of \( V \) and \( q \) versus part thickness on a three axes graph, with similar limitations as previously mentioned.

Also, as mentioned for the single control variable methods, the end thickness value that is chosen can be greater or smaller than the known reference technique thickness, but cannot be a negative entry. Similarly both \( q \) and \( V \) reference values must be positive values.

Using the Add-In Functions
The file StandXpFunc.xla needs to be placed within a suitable folder. Like all Excel Add-In files it is necessary to provide the file content for general usage within Excel by choosing “Tools > Add-Ins” and then checking the box alongside the Add-In name which should be viewable in the Add-In dialogue box. It may be necessary to use the ‘browse’ option on the dialogue box to first locate StandXpFunc.xla.

Once the Add In is available to Excel, the functions previously listed above should be seen included in the list of Excels functions (upon for example clicking the ‘function wizard’ button).

Use of the functions is like any Excel function, and each particular function requires particular input values (arguments) to calculate the required quantity. If the function wizard is invoked, these are indicated in the appropriate entry boxes within the functions dialogue box.

In the case of the array functions, then the necessary array size must first be described by making the corresponding ‘cell block’ active prior to invoking the actual function. In addition, array values are finally returned only by pressing “Ctrl + Shift +Return” together.

The details of use of the Excel Add In scalar functions and array functions is enhanced by an appreciation of the general use of Excel functions, and these details are given in

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**Figure 1:** StandardXp implementation of fixed kV approach-solution
try a variable kV

SPREADSHEET BASED APPLICATION OF THE ‘STANDARD MODEL’ IN VARIABLE PART THICKNESS RADIOGRAPHY

Appendix I. The reader is urged to trial the actual use of the ‘StandardXpFunc’ Add In functions simultaneously with study of the available explanation in Appendix I to fully appreciate and understand the nature of the functions themselves.

DISCUSSION

(i) Implementation of the Standard Model

There are several features of implementation of the standard model (of radiographic exposure technique) by the workbook application that merit discussion.

When the workbook application generates a combination solution (using both q and V as the control variables), this requires specification of two tube voltage values, one (known) value of V at the (known) reference thickness and one (chosen) value of V at the corresponding (chosen) ‘end thickness’.

The specification of these two values determines the rate of change of tube voltage with thickness (β), and consequently the effective ‘percentage increase’ per centimetre of voltage with thickness.

In the ‘variable kV approach’ according to the standard model, a five per cent (more accurately 4.7284 per cent) increase per centimetre is used. However, for a chosen range of thickness over which exposure compensation is to be made, this rate of increase may be neither desirable nor possible. Rather, since clinical technique is contrast limited, a range of voltage would apply for acceptable image contrast for a given examination type. Because of this, the implemented combination approach solution allows choice of tube voltage at the ‘end thickness’ rather than the tube charge, in determination of a list of q and V values versus thickness. In this sense the presentation of the combination solution embraces the concept of an ‘optimum kVp range’ as proposed originally by A.W. Fuchs (Fuchs, 1955).

If the entered voltage values are the same at both the reference thickness and the end thickness, then this implicitly means that the user requires no change of voltage within the technique and the solution generated is equal to that of the fixed kV method.

The functions VarqVarV and ArrVarqVarV also allow the user to obtain a solution using this combination approach. The essential calculation method used in computing the combination (V and q) approach are given in Appendix II.

For all solution methods, the displayed table of values is shown to one decimal place in the workbook application. This is essentially an academic feature of course (but practically erroneous), rather than a limitation of the application itself. The values returned from the Add-In functions will depend upon the users selected cell format. Apart from academic study it is of little value to use anything but integer values of control variables. Many X-ray generators are calibrated (at best) to +1kV at some tube voltages, and some particular X-ray generators limit the possible values of voltage that can be set by the user. Likewise there are uncertainties involved in tube charge, q, values. These facts should be borne in mind where the user is attempting to generate any exposure chart (not just by this application).

(ii) Limitations

Limitations can be considered to arise in three main contexts:

(a) Ability of the application to represent the standard model. Any limitation here is negligible in fact. Numerical accuracy is limited by the ‘parent’ application (Excel) together with the procedure code as written for this application, and any inaccuracy in generation of a final usable exposure chart is far smaller than those inaccuracies conferred by mismatch of the real system (imaged part plus radiographic system) and the standard model. This assertion is a reasonable inference because of the intrinsically high numerical accuracy of Excel code itself.

(b) Limitations of the Standard Model itself.

The limitations of the standard model itself are the major sources of inaccuracy in generating any real accurately usable exposure chart. While not the subject of this article (the accuracy of the standard model), it is prudent to highlight this fact. In particular, any method utilising a variation of tube voltage is limited in its accuracy by virtue of the photon energy spectrum response of the imaging detector.
used. The basis of the standard model does not take this into consideration.

Likewise the influence of part thickness and beam area on the secondary photon (scatter) component of the beam and the consequent variation in patient exit exposure values (let alone image density) with tube voltage is completely disregarded by the model (a constant $V^q$ proportionality of image density with tube voltage is assumed by the model, a fact which directly results in the five per cent per cm increase of tube voltage or ‘variable kV’ rule). The assumptions of the standard model are discussed in this author’s previous article (Garbett, 1999) which the reader should refer to for further details.

(There are other limitations of the standard model that could be mentioned or analysed, and these are essentially connected with the false assumptions of proportionality of image density with tube charge and proportionality of image density with a simple power law of tube voltage. Clearly, for example, image density using a fluorescent screen-film system reaches a saturated value, and so even if the standard model is taken as approximately correct, it can be approximately correct only over a limited interval of values of both $q$ and $V$.)

In any event, any exposure chart generated (by any computational means) for intended clinical use, should always be checked for suitability (check predicted control variable values at the furthest thickness values from the reference value at least), using an appropriate radiographic phantom.

This said, given that many texts still describe and advise the use of the traditional approaches to variable part thickness radiography in generating exposure charts, the application described here uses merely that same basis in determination of its displayed and listed values; that is, it uses the standard model.

In consideration of the above points, both the StandardXp workbook application and the StandXpFunc Add-In functions should not be used in a stand-alone direct clinical sense.

If used as an aid then they should be used with the same practical considerations as any other manual radiographic compensation methodology.

(c) Lack of Constraints
Apart from any limitations of the standard model, there are other reasons why the application may not give real usable solutions. Attainable tube voltage and tube charge values are limited on any X-ray generator system, and in fact these constraints are related by virtue of tube loading characteristics and the corresponding circuitry interlocks, with certain paired combinations of mAs and kV not being possible.

Likewise, any variable SID method does not take into account the non-usability of SID values where magnification effects render an image diagnostically invalid.

(A second feature might be fairly considered a criticism of the workbook application and the dedicated Add-In functions also, in that it is possible to generate SID values (L values) where the SID is smaller than the part thickness, an obvious practical problem!)

In light of the above, it is left to the user to discard those unattainable solutions that can be generated using the workbook application and the dedicated Add-In functions.

Lastly, since the application only seeks to implement the standard model, no consideration is given to ensuing patient dose arising from any generated solution values of the control variables.

The general principle of decreasing patient integral dose (also called energy imparted) for a given beam area as would result from using a larger tube voltage and smaller tube charge, should be borne in mind when trying to generate a usable exposure chart, given the over-riding constraint of diagnostic usability.

**SUMMARY**
The workbook application (StandardXp) and the (StandXpFunc) Add-In functions both allow for easy implementation of the standard models’ predictions in generating the values of exposure control variable/s required (for a constant average image density), for any part thickness. All traditional approaches are possible plus also a combination approach using variation of both tube charge and tube voltage. The application is not designed for ‘stand alone’ and direct clinical usage and is limited in this sense by the inaccuracies of the standard model itself. However, the application might be viably used as an aid to obtaining exposure technique factors if suitable practical considerations previously alerted to in the main body of this article are addressed.

The actual radiographic applicability of the predicted values thus require the user to observe the same practical and dose considerations that arise when employing any theoretical method of exposure compensation as traditionally used, and perform suitable empirical checks on a given system.

**APPENDIX I**
Each of the scalar type functions returns only the value of the control variable/s at the end thickness.

Example:
$f kV$ has arguments : $f kV(qs, xs, x_end)$ and requires the reference or ‘start’ tube charge value (in mAs) to be entered as the first argument, the value of start thickness $xs$ (in cm), entered

\[ \text{function} = \text{VarqVarV}(100,50,60,18,26) \]

\[ \text{function} = \text{VarV}(50,18,26) \]

\[ \text{function} = \text{ArrVarqVarV}(100,50,60,18,26) \]

\[ \text{function} = \text{ArrVarV}(50,18,26) \]

\[ \text{function} = \text{VarV}(50,18,26) \]

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\[ \text{function} = \text{VarqVarV}(100,50,60,18,26) \]
as the second argument, and the chosen ‘end thickness’ \( x_{\text{end}} \) entered as the third argument. Once the ‘return’ key is hit, the active Excel worksheet cell will then return the value of tube charge corresponding to the thickness \( x_{\text{end}} \).

The array functions return an array list of control variable/s versus thickness \( x \). The listed \( x \) values are limited to 5 values. These correspond to \( x_s, x_{\text{end}}, \) and quarter, mid and three quarter interval values across the range \( x_s < x < x_{\text{end}} \).

In the case of \( \text{ArrfkV}, \text{ArrVarV}, \) and \( \text{ArrVarL} \), then only the respective control variable (eg \( q \) in \( \text{ArrfkV} \)) is listed alongside the respective thickness, \( x \), value. The returned array for each of these functions is thus a 5 x 2 array. The order is \( \{C_i, x_i\}, i = 1,2,3,4,5 \) where \( C \) is the particular control variable for that function. The ‘end’ values are \( (C_5, x_5) \) and start values are \( (C_1, x_1) \) in this notation.

The function \( \text{ArrVarqVarV} \) returns a 5 x 3 array of values since both \( q \) and \( V \) are returned for a given thickness. The array list order is thus \( \{q_i, V_i, x_i\} \).

The figure below shows returned arrays for \( \text{ArrVarV} \) and \( \text{ArrVarqVarV} \).

In the case \( \text{ArrVarV} \) the value of 50kV at 18 cm is the reference technique. The chosen value for end thickness is 26 cm. Values of \( V \) are then listed at 18, 20, 22, 24 and 26 cm.

The \( \text{ArrVarqVarV} \) function below had the same initial reference values at an initial \( q = 100\text{mAs} \). The chosen end thickness voltage (end thickness also at 26 cm) was 60kV.

Also shown below each array is the single Excel cell output for the scalar functions \( \text{fkV} \) and \( \text{VarqVarV} \).

### APPENDIX II

The combination solution (altering both tube charge \( q \) and tube voltage \( V \)) as implemented in the VBA code uses the exact solutions to the system equations:

\[
\frac{dq}{dx} = (\mu - 5\beta)q \quad (1)
\]

\[
\frac{dV}{dx} = \beta V
\]

The value of the coefficient \( \mu \) is taken to be 0.231/cm. The coefficients \( \beta \) and \( (\mu - 5\beta) \) represent the rate of change of tube voltage with thickness and rate of change of tube charge with thickness respectively. Only one coefficient is freely chosen.

Specification of a starting voltage value, \( V_s \) at the reference (start) thickness \( x_s \), together with the end thickness voltage value, \( V_f \) at the ‘end thickness’ \( x_f \); determines the value of \( \beta \):

\[
ln \left( \frac{V_f}{V_s} \right) = \frac{x_f - x_s}{\beta} \quad (2)
\]

Tube charge and voltage values are then directly calculated using:

\[
q(x) = q_s \exp(\mu - 5\beta)(x - x_s) \]

\[
V(x) = V_s \exp(\beta(x - x_s)) \quad (3)
\]

where \( q \) is the value of tube charge at the reference (start) thickness \( x_s \).

Non combination solutions are generated using the relationships:

\[
q(x) = q_s \exp(\mu \Delta x) \quad (4)
\]

\[
V(x) = V_s \exp(\beta \Delta x)
\]

\[
L(x) = L_s \exp(-\gamma \Delta x)
\]

where \( \Delta x = x_f - x_s, \beta = \mu/5, \) and \( \gamma = \mu/2. \)

### REFERENCES


